150C Causal Inference

Treatment Effect Heterogeneity and Multiplicative Interaction Models

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- “Effect heterogeneity”, “Heterogeneous treatment effects,” “subgroup effects,” “interaction effects”

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- The effect of medicine X on health is positive for those below age 35, but negative for those above age 35
- Seeing negative political ads causes old people to vote, young people to stay home
- Police body cameras cause a decline in the use of force by officers in large police departments, but have no effect for officers in small police departments

Linear Interaction Model

Definition (Linear Interaction Model)

Workhorse model in social science for estimating the CATE: the linear interaction model

$$Y_i = \alpha + \beta_1 D_i + \beta_2 X_i + \beta_3 D_i * X_i + \epsilon_i$$

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- Long way: set D_i and X_i to given values, recover parameters under different scenarios.

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- $=$ Treatment effect for those units with $X = 1$ (where X could be a dummy for gender, party ID, old/young, etc.)

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- Difference between the treatment effect for those units with $X = 1$ and those units with $X = 0$
- $(\beta_1 + \beta_3) - (\beta_1) = \beta_3$
- β_3 represents the *difference* in treatment effects between the two groups (i.e. the difference-in-differences)

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- Remember "rise over run"? Change in Y given a change in X. Slope of X. All descriptions of first derivatives.

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  (Anything without X gets treated as a constant)

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- So long as $\beta_3 \neq 0$, the effect of D_i will differ depending on the value of X_i
- Multiplicative interaction model allows for heterogeneous effects

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- Standard OLS routines report standard errors for our estimates of these coefficients, $\hat{\beta}_1$ and $\hat{\beta}_3$

How to Obtain Standard Errors for Marginal Effects

Definition (Linear Interaction Model)

$$Y_i = \alpha + \beta_1 D_i + \beta_2 X_i + \beta_3 D_i * X_i + \epsilon_i$$

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- Difference in effect of D when $X = 1$ vs. when $X = 0$: β_3
- Standard OLS routines report standard errors for our estimates of these coefficients, $\hat{\beta}_1$ and $\hat{\beta}_3$
- How do we find the standard error of the marginal effect of D when X is 1, which is $\hat{\beta}_1 + \hat{\beta}_3$?

Review: Rules of Variance

Given some random variables X and Y and some constants a and b:

Definition (The Variance Operator)

$$Var[aX + bY] = a^2 Var[X] + b^2 Var[Y] + 2ab * cov[X, Y]$$

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- What are the random variables here? $\hat{\beta}_1 + \hat{\beta}_3$
- What are the constants? X
- Remember, we are estimating the uncertainty in our estimates of *coefficients* (which will vary from sample to sample due to random error) in a scenario where we are setting D and X to *constant* values (i.e. $E[Y|D = 1, X = 1]$)

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- $\sqrt{1^2 Var[\hat{\beta}_1] + X^2 Var[\hat{\beta}_3] + 2 * 1 * X * cov[\hat{\beta}_1, \hat{\beta}_3]}$
- If $X = 1$, then $SE[\hat{\beta}_1 + \hat{\beta}_3 X] = \sqrt{Var[\hat{\beta}_1] + Var[\hat{\beta}_3] + 2 * cov[\hat{\beta}_1, \hat{\beta}_3]}$
- We can compute this in-sample using estimates of the necessary variances and covariances.

Review: Variance-Covariance Matrix of Coefficients

Definition (Variance of Coefficients)

$$Var[\hat{\beta}_{OLS}] = \begin{bmatrix} Var[\hat{\beta}_1] & cov[\hat{\beta}_1, \hat{\beta}_2] & ... & cov[\hat{\beta}_1, \hat{\beta}_k] \\ cov[\hat{\beta}_2, \hat{\beta}_1] & Var[\hat{\beta}_2] & ... & cov[\hat{\beta}_2, \hat{\beta}_k] \\ \vdots & \vdots & \ddots & \vdots \\ cov[\hat{\beta}_k, \hat{\beta}_1] & cov[\hat{\beta}_k, \hat{\beta}_2] & ... & Var[\hat{\beta}_k] \end{bmatrix}$$

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- Variances on the diag, covariances on the off-diag
- Standard OLS routines estimate this matrix, and we can access it to recover $\hat{SE}[\hat{\beta}_1 + \hat{\beta}_3 X]$

Example: Gerber et al. (2015)

"Can Incarcerated Felons Be (Re)integrated into the Political System? Results from a Field Experiment". Ex-cons sent letters encouraging them to register/vote.

$$Register_i = \alpha + \beta_1 treat_i + \beta_2 Voted2008_i + \beta_3 treat_i * Voted2008_i + \epsilon_i$$

```
> summary(lm(reg ~ treat_combined+v08+ treat_combined*v08, data=d)
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.054616   0.005579   9.789  < 2e-16 ***
treat_combined     0.019151   0.007911   2.421 0.015531 *
v08                0.082999   0.024939   3.328 0.000882 ***
treat_combined:v08 0.066449   0.035049   1.896 0.058040 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1   1

Residual standard error: 0.2538 on 4333 degrees of freedom
  (2104 observations deleted due to missingness)
Multiple R-squared:  0.0129, Adjusted R-squared:  0.01221
F-statistic: 18.87 on 3 and 4333 DF,  p-value: 3.702e-12
```

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Example: Gerber et al. (2015)

Accessing $\hat{Var}[\hat{\beta}]$

```
> m<-lm(reg ~ treat_combined+v08+ treat_combined*v08, data=d)
> vc<-vcov(m)
> vc
                    (Intercept) treat_combined           v08 treat_combined:v08
(Intercept)         3.112614e-05  -3.112614e-05 -3.112614e-05       3.112614e-05
treat_combined     -3.112614e-05   6.258680e-05  3.112614e-05      -6.258680e-05
v08                -3.112614e-05   3.112614e-05  6.219516e-04      -6.219516e-04
treat_combined:v08  3.112614e-05  -6.258680e-05 -6.219516e-04       1.228412e-03
> varb1<-vc["treat_combined","treat_combined"]
> varb3<-vc["treat_combined:v08","treat_combined:v08"]
> covb1b3<-vc["treat_combined", "treat_combined:v08"]
> seb1b3<-sqrt(varb1+varb3+2*covb1b3)
> seb1b3
[1] 0.03414418
> ##95% CI
> lb<-(m$coefficients["treat_combined"]+m$coefficients["treat_combined:v08"])-2*seb1b3
> ub<-(m$coefficients["treat_combined"]+m$coefficients["treat_combined:v08"])+2*seb1b3
> lb
treat_combined
    0.01731124
> ub
treat_combined
      0.153888
```

Example: Gerber et al. (2015)

With robust standard errors

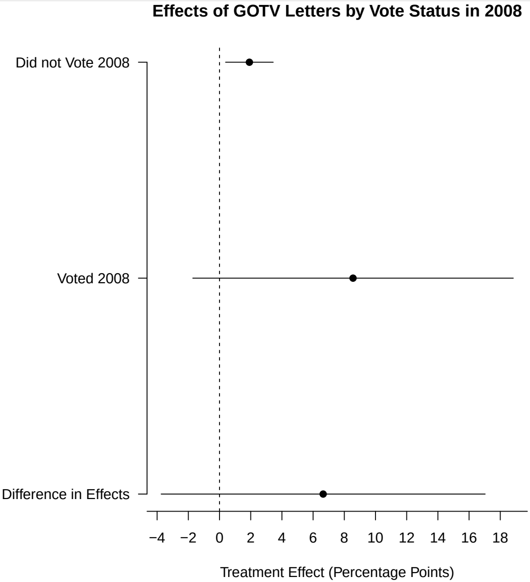
```
> vc2<-vcovHC(m, type="HC1")
> varb1<-vc2["treat_combined","treat_combined"]
> varb3<-vc2["treat_combined:v08","treat_combined:v08"]
> covb1b3<-vc2["treat_combined", "treat_combined:v08"]
> seb1b3<-sqrt(varb1+varb3+2*covb1b3)
> seb1b3
[1] 0.05137446
> ##95% CI
> lb<-(m$coefficients["treat_combined"]+m$coefficients["treat_combined:v08"])-2*seb1b3
> ub<-(m$coefficients["treat_combined"]+m$coefficients["treat_combined:v08"])+2*seb1b3
> lb
treat_combined
   -0.01714931
> ub
treat_combined
     0.1883485
```

Plotting Results

Coefficient Plots (Often better than regression tables!)

```
> coefs<-c(m$coefficients["treat_combined"], m$coefficients["treat_combined"]+
m$coefficients["treat_combined:v08"], m$coefficients["treat_combined:v08"])
> ses<-c(sqrt(varb1), seb1b3, sqrt(varb3))
> res<-cbind.data.frame(coefs=coefs, ses=ses)
> res$lb<-coefs-2*ses
> res$ub<-coefs+2*ses
>
>
> pdf(file="/Users/jonathanmummolo/Dropbox/Teaching/150C -
2017/150C2017/slides/Midterm Review/gerber_plot.pdf")
> par(mar=c(4, 8, 4, 4 ))
> y.axis<-length(coefs):1
> plot(res$coefs*100, y.axis, pch=19, cex=1, main="Effects of GOTV Letters by V
ote Status in 2008", xlim=c(min(res$lb)*100, max(res$ub)*100), axes=F,
xlab="Treatment Effect (Percentage Points)", ylab="")
> segments(res$lb*100, y.axis, res$ub*100, y.axis)
> abline(v=0, lty=2)
> axis(1, at=seq(-100, 100, by=2))
> axis(2, at=y.axis, labels=c("Did not Vote 2008", "Voted 2008", "Difference"), las=2)
> dev.off()
```

Estimates, Gerber et al. (2015)



Continuous Moderators

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$$\hat{SE}_{\hat{\beta}_1+\hat{\beta}_3} = \sqrt{\hat{Var}[\hat{\beta}_1] + X^2\hat{Var}[\hat{\beta}_3] + 2 * X * \hat{cov}[\hat{\beta}_1, \hat{\beta}_3]}$$

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Since X now takes many values besides zero and 1, the *SE* will often depend on the value of X as well!

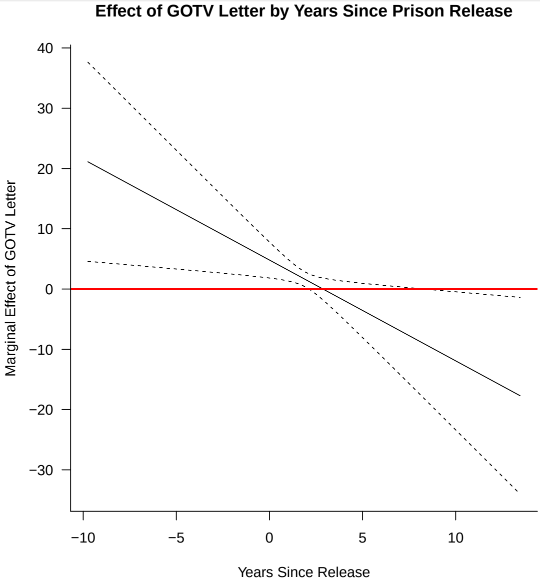
Continuous Moderators

```
> coeftest(m, vcov.=vc2)

t test of coefficients:

                                   Estimate Std. Error t value  Pr(>|t|)    
(Intercept)                       0.0631789  0.0093177  6.7805 1.304e-11 ***
treat_combined                    0.0480999  0.0149249  3.2228  0.001276 ** 
timesincerelease                 -0.0031440  0.0045173 -0.6960  0.486455    
treat_combined:timesincerelease -0.0167266  0.0070551 -2.3709  0.017776 *  
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1
```

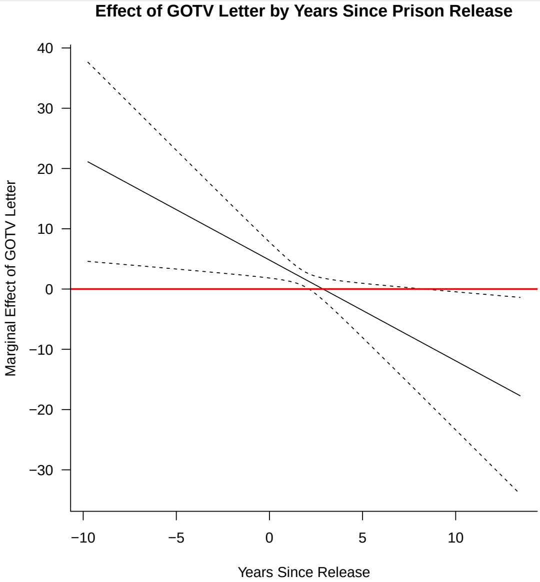
Continuous Moderators

```
> summary(d$timesincerelease)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  0.252   1.014   1.771   1.808   2.618   3.459
 x.vals<-seq(min(d$timesincerelease)-10, max(d$timesincerelease)+10, length=100)
> varb1<-vc2["treat_combined","treat_combined"]
> varb3<-vc2["treat_combined:timesincerelease","treat_combined:timesincerelease"]
> covb1b3<-vc2["treat_combined:timesincerelease", "treat_combined"]
> seb1b3<-sqrt(varb1 + x.vals^2*varb3 + 2*x.vals*covb1b3)
> seb1b3
  [1] 0.082606417 0.080957281 0.079308340 0.077659605 0.076011090 0.074362810 0.072714780
  [8] 0.071067018 0.069419543 0.067772376 0.066125540 0.064479061 0.062832966 0.061187286
 [15] 0.059542056 0.057897314 0.056253104 0.054609472 0.052966474 0.051324169 0.049682626
 [22] 0.048041924 0.046402152 0.044763412 0.043125821 0.041489516 0.039854656 0.038221424
 [29] 0.036590041 0.034960763 0.033333901 0.031709825 0.030088988 0.028471941 0.026859371
 [36] 0.025252134 0.023651317 0.022058319 0.020474964 0.018903677 0.017347735 0.015811670
 [43] 0.014301889 0.012827674 0.011402829 0.010048378 0.008796896 0.007698763 0.006828369
 [50] 0.006281113 0.006143959 0.006443150 0.007123919 0.008090510 0.009253788 0.010548882
 [57] 0.011932951 0.013378409 0.014867361 0.016387957 0.017932148 0.019494327 0.021070495
 [64] 0.022657733 0.024253866 0.025857248 0.027466609 0.029080957 0.030699505 0.032321622
 [71] 0.033946796 0.035574609 0.037204713 0.038836821 0.040470690 0.042106116 0.043742922
 [78] 0.045380961 0.047020103 0.048660237 0.050301266 0.051943104 0.053585679 0.055228923
 [85] 0.056872778 0.058517194 0.060162125 0.061807529 0.063453369 0.065099612 0.066746229
 [92] 0.068393193 0.070040478 0.071688063 0.073335929 0.074984055 0.076632425 0.078281025
 [99] 0.079929839 0.081578855
```

Estimates, Gerber et al. (2015)



Where Do We Actually Have Data?



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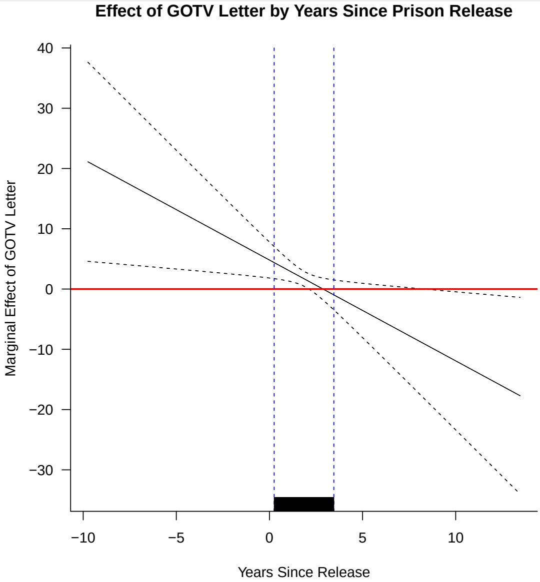
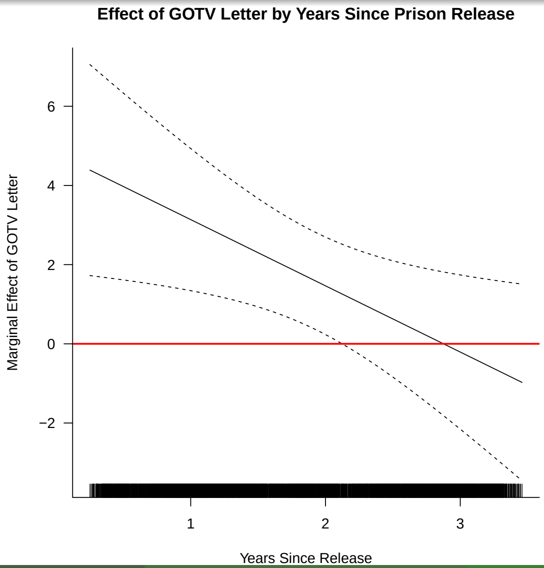


Restrict Inference to Region With Data to Avoid Extrapolation/Model Dependence



Linear Interaction Effect (LIE) Assumption

- Implicit in the linear multiplicative model: the marginal effect of $D|X$ is *linear*

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- We can use flexible estimators to allow for this and similar possibilities

Hainmueller, Mummolo and Xu (2017)

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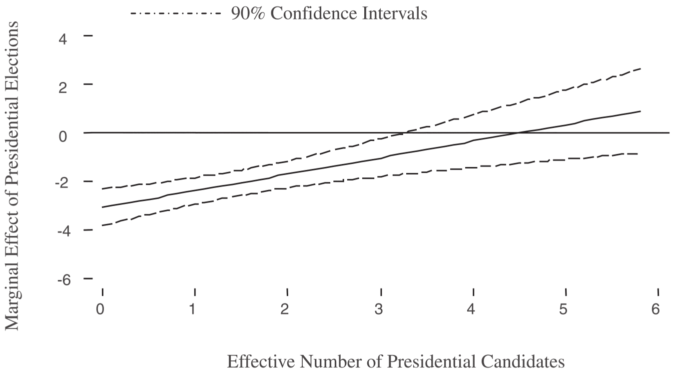
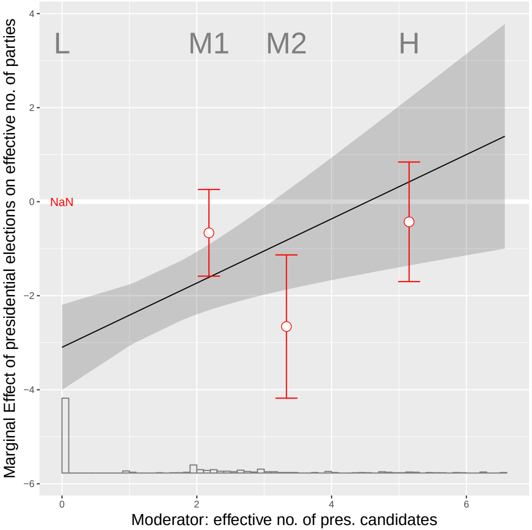
  - Look at the data! Plot joint distributions, plot marginal effect against distribution, generate cross tabs, etc.

Problem 1: Nonlinearity

Figure 2
The Marginal Effect of Temporally Proximate Presidential Elections on the Effective Number of Electoral Parties



Problem 1: Nonlinearity (Clark and Golder, 2006)



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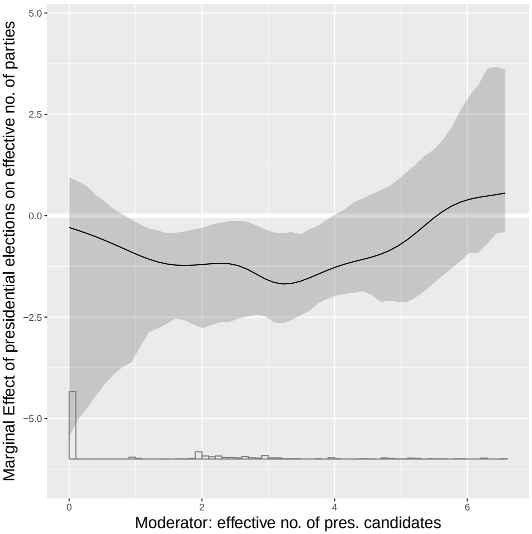
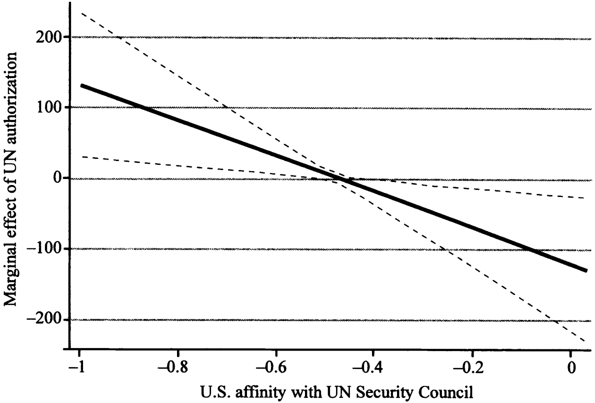


Problem 2: Extrapolation (Chapman, 2009)



Note: Dashed lines give 95 percent confidence interval.

FIGURE 2. *Marginal effect of UN authorization by affinity with the Security Council*

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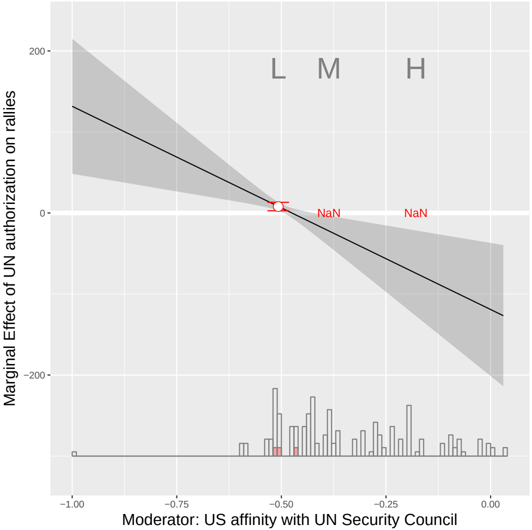

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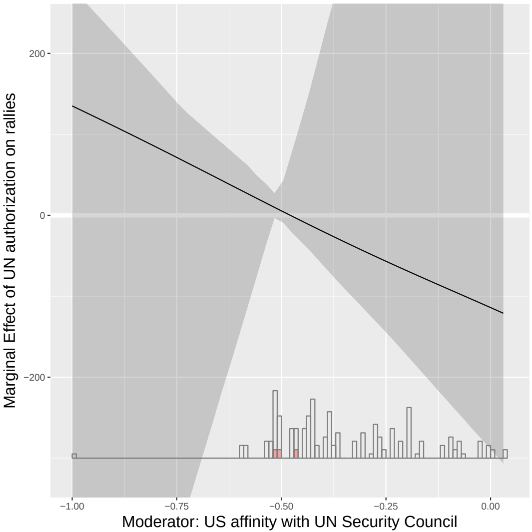


Problem 2: Extrapolation Part 2 (Nyhan and Reifler, 2010)

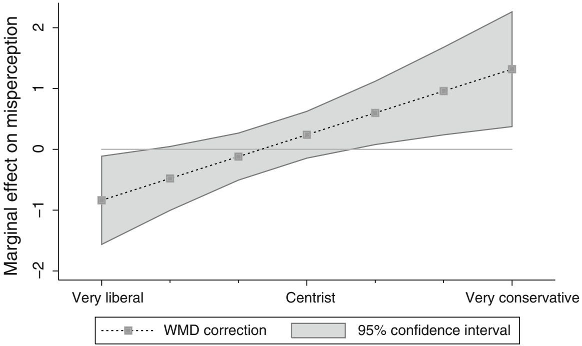


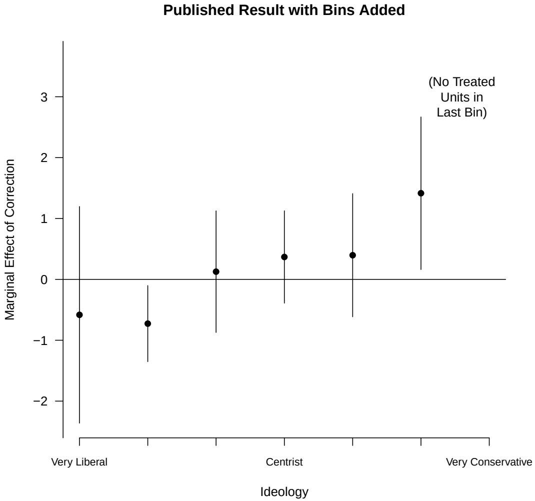
Fig. 1 Effect of correction on WMD misperception. Estimated marginal effect by ideology: fall 2005

Problem 2: Extrapolation Part 2 (Nyhan and Reifler, 2010)

Where do we have data?

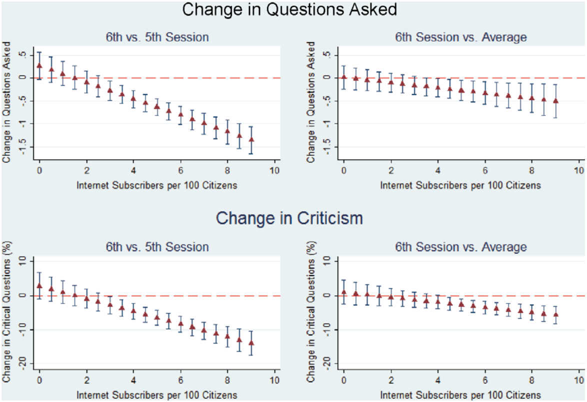
```
> dim(d)
[1] 130  13
> table(d$iraqcorr, d$ideolcen)##7 point scale

    Very liberal Liberal Somewhat left of center Centrist Somewhat right of center Conservative
  0            2      21                      10       18                       10            5
  1            3      18                       8       17                        9            5

    Very conservative
  0                 4
  1                 0
```

Problem 2: Extrapolation Part 2 (Nyhan and Reifler, 2010)



Problem 3: Interpolation (Malesky et al., 2012)

FIGURE 1. Intensity of Treatment Effect



Note: Displays the marginal effect of treatment on number of critical questions asked and percentage of critical questions, based on internet penetration, which impacts the intensity experienced by delegates. The panels are derived from the fully-specified models (4, 8, 9, and 10) in Table 5. Triangles demonstrate marginal effects. with range bars representing 90% Confidence Intervals.

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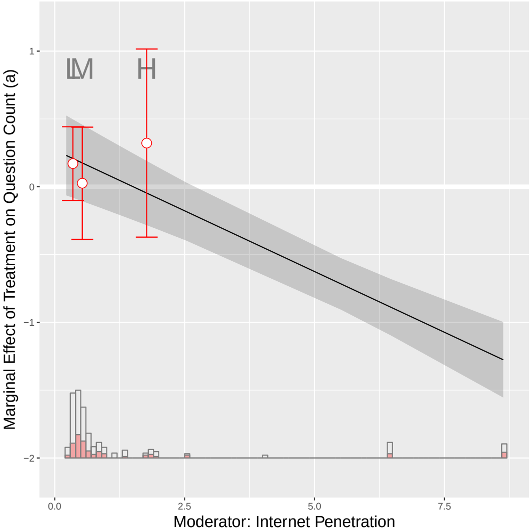

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