150C Causal Inference

Panel Methods

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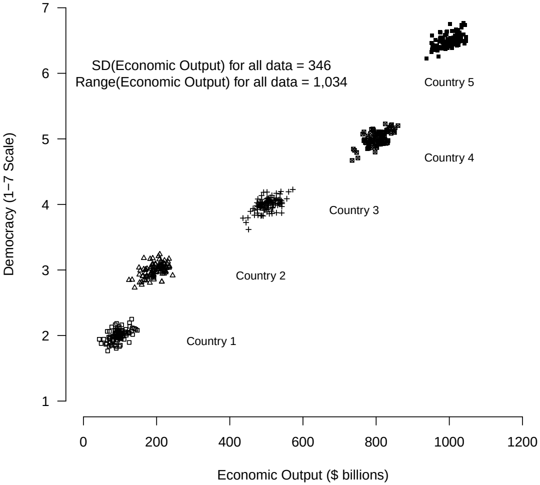
Stanford

Unobserved Group Effects



Outline

Panel Setup

- Let y and $\mathbf{x} \equiv (x_1, x_2, ..., x_K)$ be observable random variables and c be an unobservable random variable

- We are interested in the partial effects of variable x_j in the population regression function

$$E[y|x_1, x_2, ..., x_K, c]$$

- We observe a sample of $i = 1, 2, ..., N$ cross-sectional units for $t = 1, 2, ... T$ time periods (a balanced panel)
    - For each unit i, we denote the observable variables for all time periods as $\{(y_{it}, \mathbf{x}_{it}) : t = 1, 2, ..., T\}$
    - $\mathbf{x}_{it} \equiv (x_{it1}, x_{it2}, ..., x_{itK})$ is a $1 \times K$ vector

- Typically assume that cross-sectional units are i.i.d. draws from the population: $\{\mathbf{y}_i, \mathbf{x}_i, c_i\}_{i=1}^{N} \sim i.i.d.$ (cross-sectional independence)
    - $\mathbf{y}_i \equiv (y_{i1}, y_{i2}, ..., y_{iT})'$ and $\mathbf{x}_i \equiv (\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{iT})$
    - Consider asymptotic properties with T fixed and $N \to \infty$

Panel Setup

Single unit:

$$\mathbf{y}_i = \begin{pmatrix} y_{i1} \\ \vdots \\ y_{it} \\ \vdots \\ y_{iT} \end{pmatrix}_{T\times 1} \qquad \mathbf{X}_i = \begin{pmatrix} x_{i,1,1} & x_{i,1,2} & x_{i,1,j} & \cdots & x_{i,1,K} \\ \vdots & \vdots & \vdots & & \vdots \\ x_{i,t,1} & x_{i,t,2} & x_{i,t,j} & \cdots & x_{i,t,K} \\ \vdots & \vdots & \vdots & & \vdots \\ x_{i,T,1} & x_{i,T,2} & x_{i,T,j} & \cdots & x_{i,T,K} \end{pmatrix}_{T\times K}$$

Panel with all units:

$$\mathbf{y} = \begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_i \\ \vdots \\ \mathbf{y}_N \end{pmatrix}_{NT\times 1} \qquad \mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_i \\ \vdots \\ \mathbf{X}_N \end{pmatrix}_{NT\times K}$$

Outline

Unobserved Effects Model: Farm Output

- For a randomly drawn cross-sectional unit i, the model is given by

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

  - y_{it}: output of farm i in year t
  - $\mathbf{x}_{it}$: $1 \times K$ vector of variable inputs for farm i in year t, such as labor, fertilizer, etc. plus an intercept
  - $\boldsymbol{\beta}$: $K \times 1$ vector of marginal effects of variable inputs
  - c_i: farm effect, i.e. the sum of all time-invariant inputs known to farmer i (but unobserved for the researcher), such as soil quality, managerial ability, etc.
    - often called: unobserved effect, unobserved heterogeneity, etc.
  - ε_{it}: time-varying unobserved inputs, such as rainfall, unknown to the farmer at the time the decision on the variable inputs $\mathbf{x}_{it}$ is made
    - often called: idiosyncratic error
- What happens when we regress y_{it} on $\mathbf{x}_{it}$?

Pooled OLS

- When we ignore the panel structure and regress y_{it} on $\mathbf{x}_{it}$ we get

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + v_{it}, \qquad t = 1, 2, ..., T$$

  with composite error $v_{it} \equiv c_i + \varepsilon_{it}$

- Main assumption to obtain consistent estimates for $\boldsymbol{\beta}$ is:

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  with composite error $v_{it} \equiv c_i + \varepsilon_{it}$

- Main assumption to obtain consistent estimates for $\boldsymbol{\beta}$ is:
  - $E[v_{it}|\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{iT}] = E[v_{it}|\mathbf{x}_{it}] = 0$ for $t = 1, 2, ..., T$
    - $\mathbf{x}_{it}$ are strictly exogenous: the composite error v_{it} in each time period is uncorrelated with the past, current, and future regressors
    - But: labour input $\mathbf{x}_{it}$ likely depends on soil quality c_i and so we have omitted variable bias and $\hat{\boldsymbol{\beta}}$ is not consistent
  - No correlation between $\mathbf{x}_{it}$ and v_{it} implies no correlation between unobserved effect c_i and $\mathbf{x}_{it}$ for all t
    - Violations are common: whenever we omit a time-constant variable that is correlated with the regressors

Unobserved Effects Model: Program Evaluation

- Program evaluation model:

$$y_{it} = prog_{it}\,\beta + c_i + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

  - y_{it}: log wage of individual i in year t
  - $prog_{it}$: indicator coded 1 if individual i participants in training program at t and 0 otherwise
  - β: effect of program
  - c_i: sum of all time-invariant unobserved characteristics that affect wages, such as ability, etc.
- What happens when we regress y_{it} on $prog_{it}$?

Unobserved Effects Model: Program Evaluation

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  - $prog_{it}$: indicator coded 1 if individual i participants in training program at t and 0 otherwise
  - β: effect of program
  - c_i: sum of all time-invariant unobserved characteristics that affect wages, such as ability, etc.
- What happens when we regress y_{it} on $prog_{it}$? $\hat{\beta}$ not consistent since $prog_{it}$ is likely correlated with c_i (e.g. ability)
- Always ask: Is there a time-constant unobserved variable (c_i) that is correlated with the regressors? If yes, pooled OLS is problematic
- Additional problem: $v_{it} \equiv c_i + \varepsilon_{it}$ are serially correlated for same i since c_i is present in each t and thus pooled OLS standard errors are invalid

Outline

Fixed Effect Regression

- Our unobserved effects model is:

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

- If we have data on multiple time periods, we can think of c_i as fixed effects or "nuisance parameters" to be estimated

Fixed Effects Regression

Estimating a regression with the time-demeaned variables $\ddot{y}_{it} \equiv y_{it} - \bar{y}_i$ and $\ddot{\mathbf{x}}_{it} \equiv \mathbf{x}_{it} - \bar{\mathbf{x}}_i$ is numerically equivalent to a regression of y_{it} on $\mathbf{x}_{it}$ and unit specific dummy variables.

Fixed effects estimator is often called the within estimator because it only uses the time variation within each cross-sectional unit.

Even better, the regression with the time-demeaned variables is consistent for β even when $Cov[\mathbf{x}_{it}, c_i] \neq 0$, because time-demeaning eliminates the unobserved effects:

$$
\begin{aligned}
y_{it} &= \mathbf{x}_{it}\boldsymbol{\beta} + c_i + \varepsilon_{it} \\
\bar{y}_i &= \bar{\mathbf{x}}_i\boldsymbol{\beta} + c_i + \bar{\varepsilon}_i \\
\hline
(y_{it} - \bar{y}_i) &= (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)\boldsymbol{\beta} + (c_i - c_i) + (\varepsilon_{it} - \bar{\varepsilon}_i) \\
\ddot{y}_{it} &= \ddot{\mathbf{x}}_{it}\boldsymbol{\beta} + \ddot{\varepsilon}_{it}
\end{aligned}
$$

Fixed Effects Regression: Main Results

- Identification assumptions:
  1. $E[\varepsilon_{it}|\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{iT}, c_i] = 0, \ t = 1, 2, ..., T$
     - regressors are strictly exogenous conditional on the unobserved effect
     - allows $\mathbf{x}_{it}$ to be arbitrarily related to c_i
  2. regressors vary over time for at least some i and are not collinear
- Fixed effects estimator:
  1. Demean and regress $\ddot{y}_{it}$ on $\ddot{\mathbf{x}}_{it}$ (need to correct degrees of freedom)
  2. Regress y_{it} on $\mathbf{x}_{it}$ and unit dummies (dummy variable regression)
  3. Regress y_{it} on $\mathbf{x}_{it}$ with canned fixed effects routine
     - R: `plm(y~x , model = within, data = data)`
- Properties (under assumptions 1-2):
  - $\hat{\boldsymbol{\beta}}_{FE}$ is consistent: $\underset{N\rightarrow\infty}{\text{plim}}\ \hat{\boldsymbol{\beta}}_{FE,N} = \boldsymbol{\beta}$
  - $\hat{\boldsymbol{\beta}}_{FE}$ also unbiased conditional on $\mathbf{X}$

Fixed Effects Regression: Main Issues

- Inference:
  - Standard errors have to be "clustered" by panel unit (e.g. farm) to allow correlation in the ε_{it}'s for the same i.
    - R: `coeftest(mod, vcov=function(x) vcovHC(x, cluster="group", type="HC1"))`
  - Yields valid inference as long as number of clusters is reasonably large
- Typically we care about $\boldsymbol{\beta}$, but unit fixed effects c_i could be of interest
  - `plm` routine demeans the data before running the regression and therefore does not estimate $\widehat{c}_i$

Example: Direct Democracy and Naturalizations

- Do minorities fare worse under direct democracy than under representative democracy?
- Hainmueller and Hangartner (2016, AJPS) examine data on naturalization requests of immigrants in Switzerland, where municipalities vote on naturalization applications in:
  - referendums (direct democracy)
  - elected municipality councils (representative democracy)
- Annual panel data from 1,400 municipalities for the 1991-2009 period
  - y_{it}: naturalization rate =
    $\#$ naturalizations$_{it}$ / eligible foreign population $_{it-1}$
  - x_{it}: 1 if municipality used representative democracy, 0 if municipality used direct democracy in year t

Naturalization Panel Data Long Format

```
> d <- read.dta("Swiss_Panel_long.dta")
> print(d[30:40,],digits=2)
   muniID      muni_name year nat_rate repdem
30      2 Affoltern A.A. 2001     3.21      0
31      2 Affoltern A.A. 2002     4.64      0
32      2 Affoltern A.A. 2003     4.84      0
33      2 Affoltern A.A. 2004     5.62      0
34      2 Affoltern A.A. 2005     4.39      0
35      2 Affoltern A.A. 2006     8.12      1
36      2 Affoltern A.A. 2007     7.07      1
37      2 Affoltern A.A. 2008     8.98      1
38      2 Affoltern A.A. 2009     6.12      1
39      3     Bonstetten 1991     0.83      0
40      3     Bonstetten 1992     0.84      0
```

Pooled OLS

```
> summary(lm(nat_rate~repdem,data=d))

Call:
lm(formula = nat_rate ~ repdem, data = d)

Residuals:
   Min      1Q  Median      3Q     Max
-4.726 -2.223 -1.523  1.411 21.915

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.22268    0.06904   32.19   <2e-16 ***
repdem       2.50332    0.12907   19.39   <2e-16 ***
```

Time-Demeaning for Fixed Effects: $y_{it} \rightarrow \ddot{y}_{it}$

```
> library(plyr)
> d <- ddply(d, .(muniID), transform,
+                        nat_rate_demean = nat_rate - mean(nat_rate),
+                        nat_rate_mean   = mean(nat_rate),
+                        repdem_demean   = repdem - mean(repdem))
>
> print(d[20:38,
+       c("muniID","muni_name","year","nat_rate","nat_rate_mean","nat_rate_demean","repdem","repdem_demean")
+       ],digits=2)
   muniID      muni_name year nat_rate nat_rate_mean nat_rate_demean repdem repdem_demean
20      2 Affoltern A.A. 1991     0.22           3.6           -3.38      0         -0.21
21      2 Affoltern A.A. 1992     0.95           3.6           -2.65      0         -0.21
22      2 Affoltern A.A. 1993     1.05           3.6           -2.55      0         -0.21
23      2 Affoltern A.A. 1994     0.83           3.6           -2.76      0         -0.21
24      2 Affoltern A.A. 1995     2.00           3.6           -1.59      0         -0.21
25      2 Affoltern A.A. 1996     1.78           3.6           -1.82      0         -0.21
26      2 Affoltern A.A. 1997     1.86           3.6           -1.73      0         -0.21
27      2 Affoltern A.A. 1998     2.05           3.6           -1.54      0         -0.21
28      2 Affoltern A.A. 1999     2.40           3.6           -1.19      0         -0.21
29      2 Affoltern A.A. 2000     2.20           3.6           -1.40      0         -0.21
30      2 Affoltern A.A. 2001     3.21           3.6           -0.39      0         -0.21
31      2 Affoltern A.A. 2002     4.64           3.6            1.04      0         -0.21
32      2 Affoltern A.A. 2003     4.84           3.6            1.25      0         -0.21
33      2 Affoltern A.A. 2004     5.62           3.6            2.03      0         -0.21
34      2 Affoltern A.A. 2005     4.39           3.6            0.79      0         -0.21
35      2 Affoltern A.A. 2006     8.12           3.6            4.52      1          0.79
36      2 Affoltern A.A. 2007     7.07           3.6            3.47      1          0.79
37      2 Affoltern A.A. 2008     8.98           3.6            5.38      1          0.79
38      2 Affoltern A.A. 2009     6.12           3.6            2.52      1          0.79
```

Fixed Effects Regression with Demeaned Data

```
> summary(lm(nat_rate_demean~repdem_demean,data=d))

Call:
lm(formula = nat_rate_demean ~ repdem_demean, data = d)

Residuals:
    Min      1Q  Median      3Q     Max
-8.4712 -2.0883 -0.5978  1.0841 21.3076

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.266e-16  5.279e-02    0.00        1
repdem_demean 3.023e+00  1.293e-01   23.39   <2e-16 ***
```

Fixed Effects Regression with Canned Routine

```
> library(plm)
> library(lmtest)
> d <- plm.data(d, indexes = c("muniID", "year"))
> mod_fe <- plm(nat_rate~repdem,data=d,model="within")
> coeftest(mod_fe,
vcov=function(x) vcovHC(x, cluster="group", type="HC1"))

t test of coefficients:

       Estimate Std. Error t value  Pr(>|t|)
repdem  3.02280    0.18525  16.318 < 2.2e-16 ***
---
```

Fixed Effects Regression with Dummies

```
> mod_du <- plm(nat_rate~repdem+as.factor(muniID),data=d,model="pooling")
> coeftest(mod_du, vcov=function(x) vcovHC(x, cluster="group", type="HC1"))

t test of coefficients:

                       Estimate  Std. Error     t value  Pr(>|t|)
(Intercept)          1.5922e+00  4.0068e-02  3.9737e+01 < 2.2e-16 ***
repdem               3.0228e+00  1.9032e-01  1.5883e+01 < 2.2e-16 ***
as.factor(muniID)2   1.3674e+00  1.4249e-08  9.5960e+07 < 2.2e-16 ***
as.factor(muniID)3   1.2923e+00  1.4283e-08  9.0472e+07 < 2.2e-16 ***
as.factor(muniID)9   1.2847e+00  1.3404e-08  9.5837e+07 < 2.2e-16 ***
as.factor(muniID)10  1.2718e+00  1.4182e-08  8.9675e+07 < 2.2e-16 ***
as.factor(muniID)13  3.2655e-01  1.2597e-08  2.5922e+07 < 2.2e-16 ***
as.factor(muniID)25  5.6413e-02  3.0051e-02  1.8772e+00 0.0605523 .
as.factor(muniID)26  3.1257e+00  1.0017e-02  3.1204e+02 < 2.2e-16 ***
as.factor(muniID)29  3.1797e+00  3.0051e-02  1.0581e+02 < 2.2e-16 ***
as.factor(muniID)33  3.2293e+00          NA          NA        NA
as.factor(muniID)34  1.7467e+00  3.0051e-02  5.8123e+01 < 2.2e-16 ***
.
```

Applying Fixed Effects

- We can use fixed effects for other data structures to restrict comparisons to within unit variation
  - Matched pairs
    - Twin fixed effects to control for unobserved effects of family background
  - Cluster fixed effects in hierarchical data
    - School fixed effects to control for unobserved effects of school

Problems that (even) fixed effects do not solve

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

- Where y_{it} is murder rate and x_{it} is police spending per capita
- What happens when we regress y on x and city fixed effects?

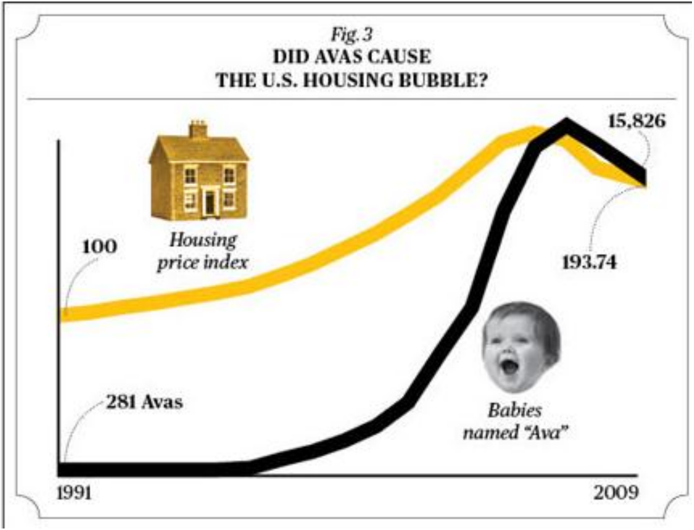
Problems that (even) fixed effects do not solve

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

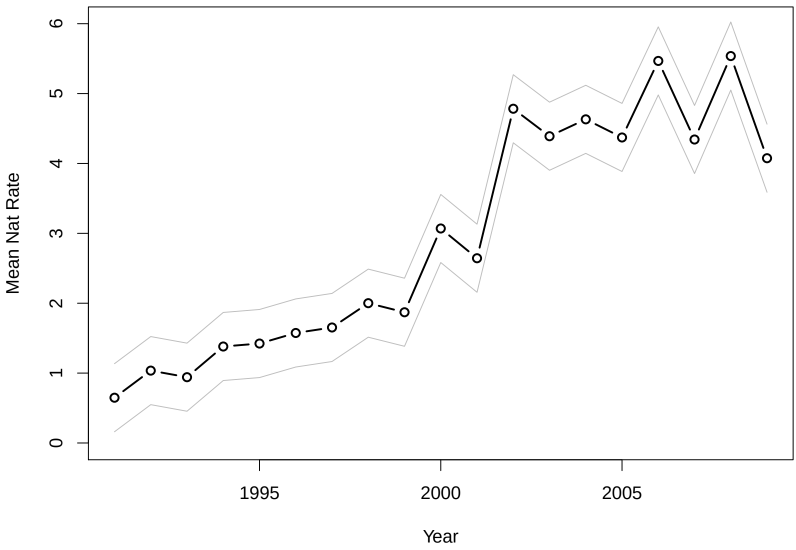
- Where y_{it} is murder rate and x_{it} is police spending per capita
- What happens when we regress y on x and city fixed effects?
  - $\hat{\boldsymbol{\beta}}_{FE}$ inconsistent unless strict exogeneity conditional on c_i holds
    - $E[\varepsilon_{it}|\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{iT}, c_i] = 0, \ t = 1, 2, ..., T$
    - implies ε_{it} uncorrelated with past, current, and future regressors
- Most common violations:
  1. Time-varying omitted variables
     - economic boom leads to more police spending and less murders
     - can include time-varying controls, but avoid post-treatment bias
  2. Simultaneity
     - if city adjusts police based on past murder rate, then spending$_{t+1}$ is correlated with ε_t (since higher ε_t leads to higher murder rate at t)
     - strictly exogenous x cannot react to what happens to y in the past or the future!
- Fixed effects do not obviate need for good research design!

Outline

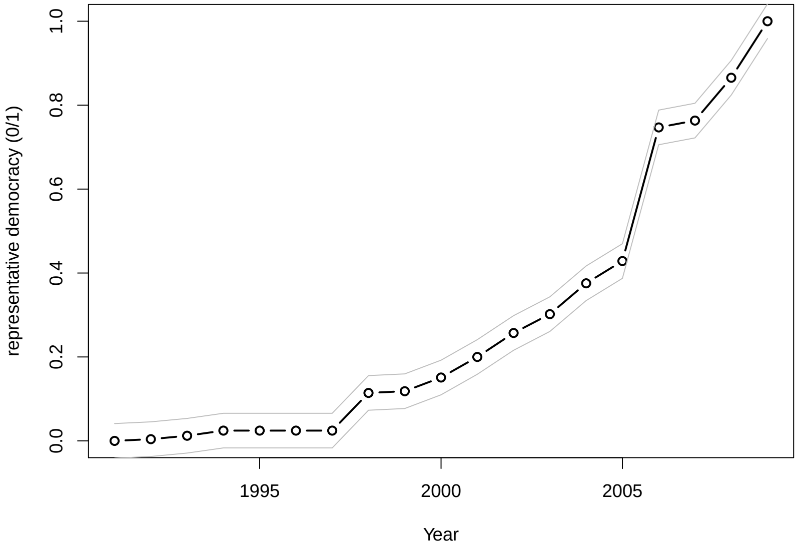
Common Shocks or Causation?



Naturalization Rate Over Time



Representative Democracy Over Time



Adding Time Effects

- Reconsider our unobserved effects model:

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

  - Fixed effects assumption: $E[\varepsilon_{it}|\mathbf{x}_i, c_i] = 0,\ \ t = 1, 2, ..., T$: regressors are strictly exogenous conditional on the unobserved effect
  - Typical violation: Common shocks that affect all units in the same way and are correlated with $\mathbf{x}_{it}$.
    - Trends in farming technology or climate affect productivity
    - Trends in immigration inflows affect naturalization rates
  - We can allow for such common shocks by including time effects into the model

Fixed Effects Regression: Adding Time Effects

- Linear time trend:

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + t + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

  - Linear time trend common to all units

- Time fixed effects:

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + t_t + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

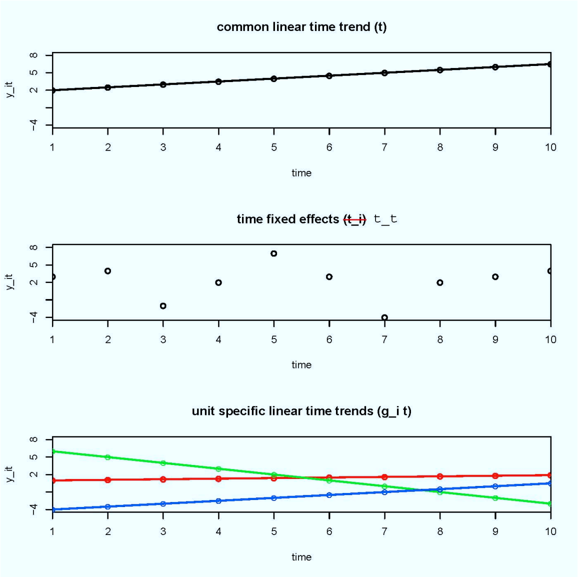
  - Common shock in each time period
  - Generalized difference-in-differences model

- Unit specific linear time trends:

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + g_i \cdot t + t_t + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

  - Linear time trends that vary by unit

Modeling Time Effects



Modeling Time Effects

```
> d$time <- as.numeric(d$year)
> d[1:21,c("muniID","muni_name","year","time")]
   muniID      muni_name year time
1       1    Aeugst A.A. 1991    1
2       1    Aeugst A.A. 1992    2
3       1    Aeugst A.A. 1993    3
4       1    Aeugst A.A. 1994    4
5       1    Aeugst A.A. 1995    5
6       1    Aeugst A.A. 1996    6
7       1    Aeugst A.A. 1997    7
8       1    Aeugst A.A. 1998    8
9       1    Aeugst A.A. 1999    9
10      1    Aeugst A.A. 2000   10
11      1    Aeugst A.A. 2001   11
12      1    Aeugst A.A. 2002   12
13      1    Aeugst A.A. 2003   13
14      1    Aeugst A.A. 2004   14
15      1    Aeugst A.A. 2005   15
16      1    Aeugst A.A. 2006   16
17      1    Aeugst A.A. 2007   17
18      1    Aeugst A.A. 2008   18
19      1    Aeugst A.A. 2009   19
20      2 Affoltern A.A. 1991    1
21      2 Affoltern A.A. 1992    2
```

Fixed Effects Regression: Linear Time Trend

```
> mod_fe <- plm(nat_rate~repdem+time,data=d,model="within")
> coeftest(mod_fe, vcov=function(x)
vcovHC(x, cluster="group", type="HC1"))

t test of coefficients:

       Estimate Std. Error t value  Pr(>|t|)
repdem  0.82479    0.25853  3.1903  0.001431 **
time    0.23137    0.01714 13.4987 < 2.2e-16 ***
```

Fixed Effects Regression: Year Fixed Effects

```
> mod_fe <- plm(nat_rate~repdem+year,data=d,model="within")
> coeftest(mod_fe, vcov=function(x) vcovHC(x, cluster="group", type="HC1"))

t test of coefficients:

          Estimate Std. Error t value  Pr(>|t|)
repdem     1.20366    0.30253  3.9786 7.043e-05 ***
year1992   0.38292    0.17197  2.2266   0.02602 *
year1993   0.27898    0.15110  1.8463   0.06492 .
year1994   0.70341    0.16712  4.2089 2.617e-05 ***
year1995   0.74591    0.17827  4.1841 2.919e-05 ***
year1996   0.89693    0.18345  4.8892 1.049e-06 ***
year1997   0.97570    0.18661  5.2285 1.788e-07 ***
year1998   1.21550    0.22506  5.4007 6.988e-08 ***
year1999   1.08051    0.21430  5.0419 4.794e-07 ***
year2000   2.23993    0.23968  9.3457 < 2.2e-16 ***
year2001   1.75531    0.24790  7.0807 1.662e-12 ***
year2002   3.82573    0.32672 11.7096 < 2.2e-16 ***
year2003   3.37837    0.32664 10.3428 < 2.2e-16 ***
year2004   3.53176    0.34285 10.3012 < 2.2e-16 ***
year2005   3.20837    0.31097 10.3171 < 2.2e-16 ***
year2006   3.92057    0.39023 10.0468 < 2.2e-16 ***
year2007   2.77646    0.36884  7.5276 6.237e-14 ***
year2008   3.84780    0.40135  9.5872 < 2.2e-16 ***
year2009   2.22388    0.41997  5.2953 1.246e-07 ***
---
```

Unit Specific Time Trends Often Eliminate "Results"

TABLE 5.2.3
Estimated effects of labor regulation on the performance of firms in Indian states

	(1)	(2)	(3)	(4)
Labor regulation (lagged)	−.186	−.185	−.104	.0002
	(.064)	(.051)	(.039)	(.020)
Log development expenditure per capita		.240	.184	.241
		(.128)	(.119)	(.106)
Log installed electricity capacity per capita		.089	.082	.023
		(.061)	(.054)	(.033)
Log state population		.720	0.310	−1.419
		(.96)	(1.192)	(2.326)
Congress majority			−.0009	.020
			(.01)	(.010)
Hard left majority			−.050	−.007
			(.017)	(.009)
Janata majority			.008	−.020
			(.026)	(.033)
Regional majority			.006	.026
			(.009)	(.023)
State-specific trends	No	No	No	Yes
Adjusted R^2	.93	.93	.94	.95

Notes: Adapted from Besley and Burgess (2004), table IV. The table reports regression DD estimates of the effects of labor regulation on productivity. The

"labor regulation increased in states where output was declining anyway"

Fixed Effects Regression: Unit Specific Time Trends

```
> mod_fe <- plm(nat_rate~
repdem+muniID*time+year,data=d,model="within")
> coeftest(mod_fe, vcov=function(x)
vcovHC(x, cluster="group", type="HC1"))

t test of coefficients:

                  Estimate   Std. Error     t value   Pr(>|t|)
repdem           0.9865241    .322868     3.0634545  2.043e-05 ***
muniID2:time   -4.1916e-02  2.0515e-09 -2.0432e+07 < 2.2e-16 ***
muniID3:time   -8.4358e-02  2.1145e-09 -3.9896e+07 < 2.2e-16 ***
.
```

Unit Specific Quadratic Time Trends

```
> d$time2 <- d$time^2
> mod_fe <- plm(nat_rate~repdem+
muniID*time+muniID*time^2+year,data=d,model="within")
> coeftest(mod_fe, vcov=function(x)
vcovHC(x, cluster="group", type="HC1"))

t test of coefficients:

                   Estimate   Std. Error      t value   Pr(>|t|)
repdem           1.22272779    .3804359   3.212323  1.023e-05 ***
muniID2:time     1.37177084  1.034e-09     .0344+07 < 3.4e-16 ***
muniID2:time2   -0.07068432  2.2034e-09 -1.234e+07 < 5.6e-16 ***
.
```

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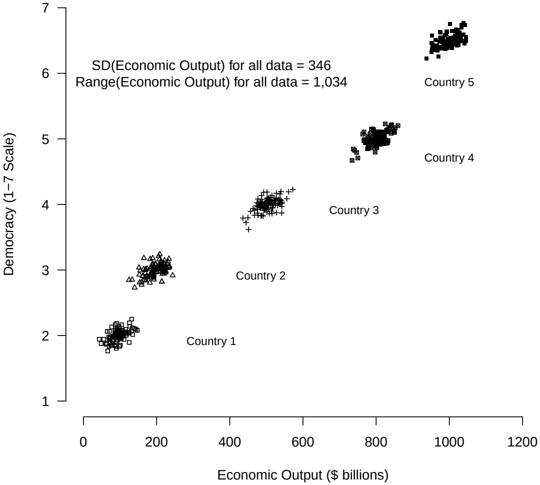
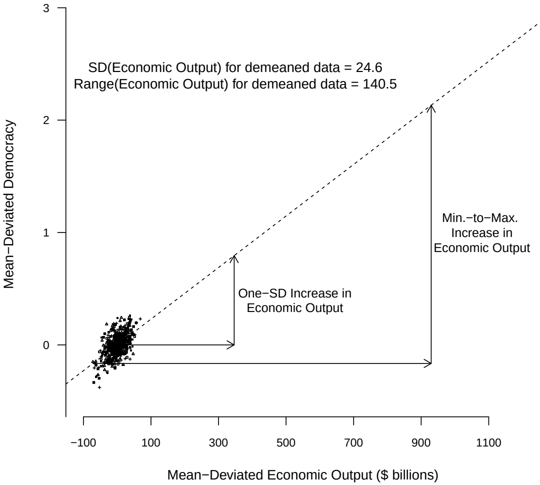
- More important when treatment is continuous
- Useful first step: examine (plot, summarize) the demeaned data
- Also a good way to check functional form assumptions: is the bivariate relationship of interest still linear after conditioning on covariates?

Pooled Data


SD(Economic Output) for all data = 346
Range(Economic Output) for all data = 1,034
Country 1
Country 2
Country 3
Country 4
Country 5
Democracy (1–7 Scale)
Economic Output ($ billions)
1
2
3
4
5
6
7
0
200
400
600
800
1000
1200

Reduction in Variation



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```
> head(d[,c("name_state","county","year",
"all_current_relief","incum_vote")])
  name_state  county year all_current_relief incum_vote
1         AL AUTAUGA 1988           0.000000   67.12975
2         AL AUTAUGA 1992           0.473335   55.92000
3         AL AUTAUGA 1996           0.000000   32.52000
4         AL AUTAUGA 2000           0.000000   28.72000
5         AL AUTAUGA 2004           4.257831   75.67000
6         AL BALDWIN 1988           3.323777   72.84960
```

Example: Healy and Malhotra (2009)

- Pooled OLS Result

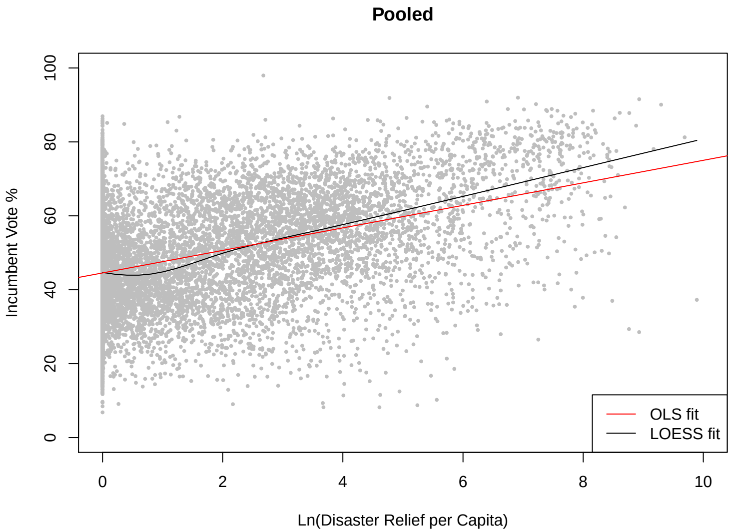
```
> summary(lm(incum_vote~all_current_relief, data=d))

Call:
lm(formula = incum_vote ~ all_current_relief, data = d)

Residuals:
    Min      1Q  Median      3Q     Max
-51.761  -8.551  -0.212   8.618  45.258

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        44.55219    0.11849  376.00   <2e-16 ***
all_current_relief  3.04719    0.05529   55.12   <2e-16 ***
```

Bivariate Plot

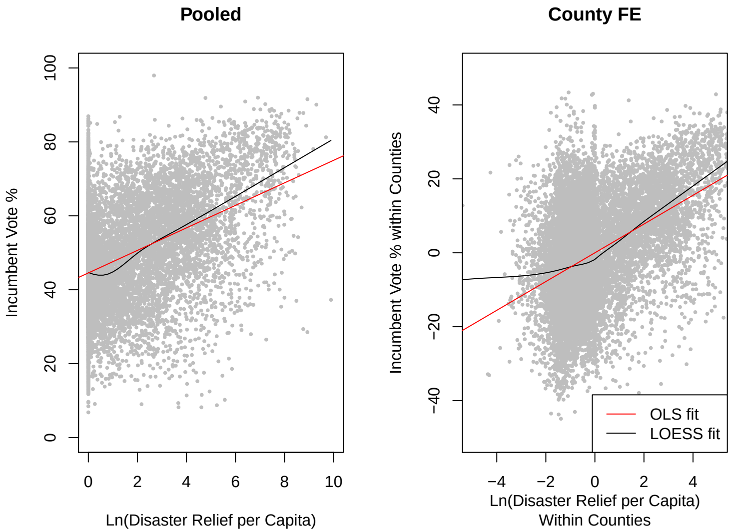


Example: Healy and Malhotra (2009)

- County Fixed Effects

```
> summary(m1<-lm(incum_vote~ all_current_relief
+factor(fips), data=d))$coefficients[1:2,]
                    Estimate Std. Error  t value     Pr(>|t|)
(Intercept)         48.31139 5.99207392  8.06255 8.152436e-16
all_current_relief   3.88969 0.06523738 59.62364 0.000000e+00
```

County FE


Pooled
County FE
Incumbent Vote %
100
80
60
40
20
0
0
2
4
6
8
10
Ln(Disaster Relief per Capita)
Incumbent Vote % within Counties
40
20
0
−20
−40
−4
−2
0
2
4
Ln(Disaster Relief per Capita)
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OLS fit
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What About Multi-Way Fixed Effects?

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- How do we visualize data with multi-way fixed effects?
- Can “residualize” the data and then plot
- Useful exercise with all multivariate regressions

Frisch-Waugh-Lovell Theorem

Suppose we are interested in estimating β_1 in the following model:

$$y = \alpha + \beta_1 x + \delta_2 z_1 + \beta_3 z_2 + ... + \beta_k z_k + \varepsilon$$

where x is the treatment and z_1-z_k are a set of control variables. β_1 can also be estimated through the following multi-step process.

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In one-way FE only, equivalent to demeaning the data within each cross-sectional unit

Healy and Malhotra (2009)

1. Regress Disaster Relief on an intercept and county and year dummies, and store the residuals, $r_{xz} = (x - \hat{x}_{xz})$

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1. Regress Disaster Relief on an intercept and county and year dummies, and store the residuals, $r_{xz} = (x - \hat{x}_{xz})$
2. Regress Vote Share on an intercept and county and year dummies and store the residuals, $r_{yz} = (y - \hat{y}_{yz})$
3. Estimate $r_{yz} = \theta + \beta_1^* r_{xz} + v$

County and Year FE

Dummy Model:

```
> m1b<-lm(incum_vote~ all_current_relief+
factor(fips)+factor(year), data=d)
> summary(m1b)$coefficients[1:5,]
                     Estimate Std. Error     t value      Pr(>|t|)
(Intercept)        59.0045324 5.18832585 11.37255717  8.039495e-30
all_current_relief  2.1673932 0.09253326 23.42285543 9.099120e-119
factor(fips)1003   -2.5218028 7.33301005 -0.34389736  7.309293e-01
factor(fips)1005   -4.1090401 7.33462683 -0.56022484  5.753362e-01
factor(fips)1007    0.1012718 7.33254821  0.01381127  9.889808e-01
```

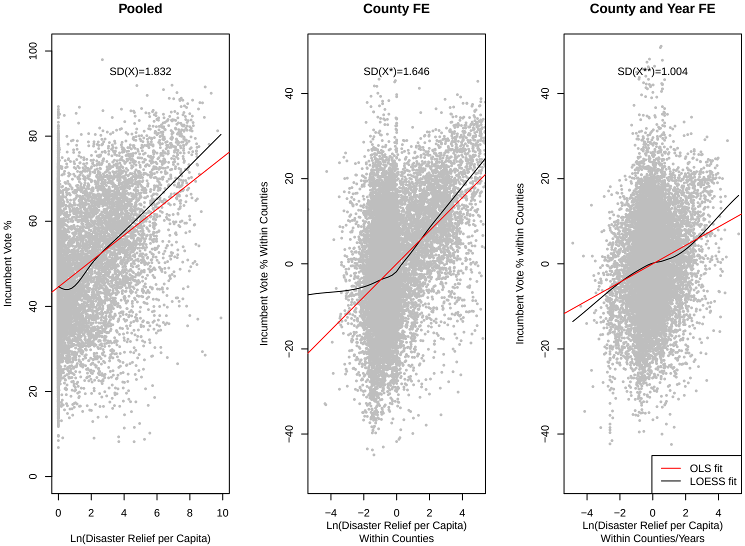
County and Year FE

Residualized Model:

```
m3b<-lm(all_current_relief~factor(fips)+factor(year), data=d)
m4b<-lm(incum_vote ~factor(fips)+factor(year), data=d)

> summary(lm(m4b$residuals~m3b$residuals))$coefficients
                  Estimate Std. Error      t value      Pr(>|t|)
(Intercept)   6.219470e-16 0.08310859 7.483547e-15  1.000000e+00
m3b$residuals 2.167393e+00 0.08274764 2.619281e+01 5.088501e-148
```

County and Year FE



Interpreting Results

Dummy Model:

```
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> summary(m1b)$coefficients[1:5,]
                      Estimate Std. Error     t value      Pr(>|t|)
(Intercept)         59.0045324 5.18832585 11.37255717  8.039495e-30
all_current_relief   2.1673932 0.09253326 23.42285543 9.099120e-119
```

- With county and year FE, more reasonable to discuss a one-SD shift in X^{**} = disaster relief after residualizing with respect to county and year dummies
- 1.044 * 2.17 = 2.26 additional points for incumbent using SD of X^{**}
- 1.832 * 2.17 = 3.97 additional points for incumbent using SD of X
- $\frac{2.26}{3.97}$ = .569 meaning the substantive impact is cut in half once we consider a plausible shift in X.

Outline

Leads and Lags

- Let D_{it} be a binary indicator coded 1 if unit i switched from control to treatment between t and $t-1$; 0 otherwise
  - Lags: D_{it-1}: unit switched between $t-1$ and $t-2$
  - Leads: D_{it+1}: unit switches between $t+1$ and t
- Include lags and leads into the fixed effects model:

$$y_{it} = D_{it+2}\boldsymbol{\beta}_{-2} + D_{it+1}\boldsymbol{\beta}_{-1} + D_{it}\boldsymbol{\beta}_{0} + D_{it-1}\boldsymbol{\beta}_{1} + D_{it-2}\boldsymbol{\beta}_{2} + c_i + \varepsilon_{it}$$

- Interpretation of coefficients:
  - Leads $\boldsymbol{\beta}_{-1}$, $\boldsymbol{\beta}_{-2}$, etc. test for anticipation effects (should be zero!)
  - Switch $\boldsymbol{\beta}_0$ tests for immediate effect
  - Lags $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, etc. test for long-run effects
    - highest lag dummy can be coded 1 for all post-switch years

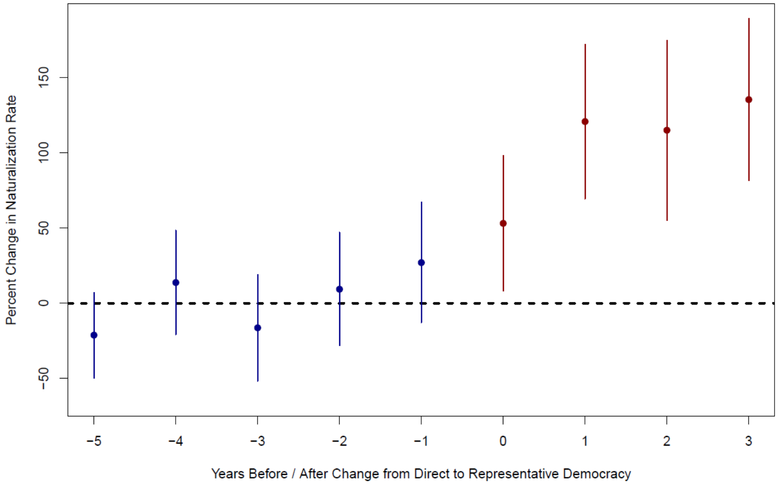
Lags and Leads of Switch to Representative Democracy

```
> d[970:989,c(1:3,5,12:ncol(d))]
    muniID year muni_name repdem switcht lag1 lag2 lag3 lead1 lead2 lead3 lead4 lead5
970    220 1991 Hagenbuch      0       0    0    0    0     0     0     0     0     0
971    220 1992 Hagenbuch      0       0    0    0    0     0     0     0     0     0
972    220 1993 Hagenbuch      0       0    0    0    0     0     0     0     0     0
973    220 1994 Hagenbuch      0       0    0    0    0     0     0     0     0     0
974    220 1995 Hagenbuch      0       0    0    0    0     0     0     0     0     0
975    220 1996 Hagenbuch      0       0    0    0    0     0     0     0     0     0
976    220 1997 Hagenbuch      0       0    0    0    0     0     0     0     0     0
977    220 1998 Hagenbuch      0       0    0    0    0     0     0     0     0     1
978    220 1999 Hagenbuch      0       0    0    0    0     0     0     0     1     0
979    220 2000 Hagenbuch      0       0    0    0    0     0     0     1     0     0
980    220 2001 Hagenbuch      0       0    0    0    0     0     1     0     0     0
981    220 2002 Hagenbuch      0       0    0    0    0     1     0     0     0     0
982    220 2003 Hagenbuch      1       1    0    0    0     0     0     0     0     0
983    220 2004 Hagenbuch      1       0    1    0    0     0     0     0     0     0
984    220 2005 Hagenbuch      1       0    0    1    0     0     0     0     0     0
985    220 2006 Hagenbuch      1       0    0    0    1     0     0     0     0     0
986    220 2007 Hagenbuch      1       0    0    0    1     0     0     0     0     0
987    220 2008 Hagenbuch      1       0    0    0    1     0     0     0     0     0
988    220 2009 Hagenbuch      1       0    0    0    1     0     0     0     0     0
989    224 1991   Pfungen      0       0    0    0    0     0     0     0     0     0
```

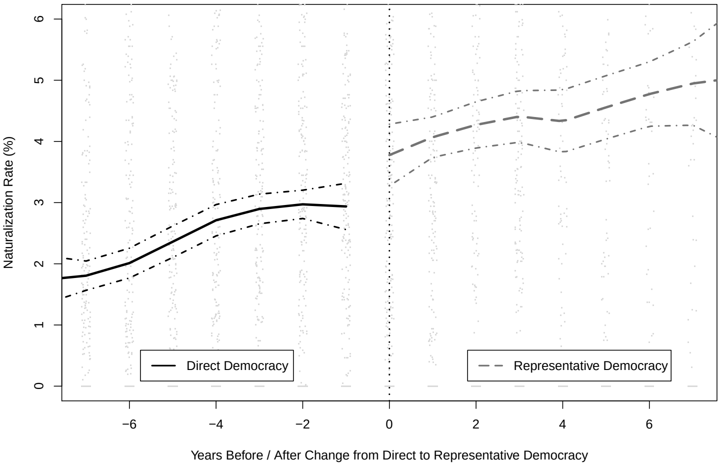
Dynamic Effect of Switching to Representative Democracy

```
> mod_all    <- plm(nat_rate~lag3+lag2+lag1+switcht+
lead1+lead2+lead3+lead4+lead5+
+                          year,data=d,model="within")
> coeftest(mod_all, vcov=function(x)
vcovHC(x, cluster="group", type="HC1"))

t test of coefficients:

          Estimate Std. Error t value  Pr(>|t|)
lag3      1.160345   0.506989  2.2887 0.0221442 *
lag2      1.743682   0.538419  3.2385 0.0012105 **
lag1      1.881663   0.487013  3.8637 0.0001133 ***
switcht   0.756479   0.427751  1.7685 0.0770463 .
lead1     0.213876   0.389191  0.5495 0.5826635
lead2     0.084368   0.356799  0.2365 0.8130891
lead3     0.144045   0.318756  0.4519 0.6513661
lead4     0.075019   0.298425  0.2514 0.8015287
lead5    -0.094241   0.259448 -0.3632 0.7164439
year1992  0.385289   0.172172  2.2378 0.0252829 *
```

Dynamic Effect of Switching to Representative Democracy



Switching Plot



Heterogeneous Treatment Effects

- So far we have assumed that the treatment effect is constant across units
- Can allow for heterogeneous treatment effects by including interaction of treatment with other regressors

$$y_{it} = treat_{it}\alpha_0 + (treat_{it} \cdot x_{it})\alpha_1 + x_{it}\boldsymbol{\beta} + c_i + t + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

- Often the treatment is interacted with a time-invariant regressor:

$$y_{it} = treat_{it}\alpha_0 + (treat_{it} \cdot x_i)\alpha_1 + c_i + t + \varepsilon_{it}, \qquad t = 1, 2, ..., T$$

  - Note: The lower order term on the time-invariant x_i is collinear with the fixed effects and drops out

Heterogeneous Effect of Direct Democracy